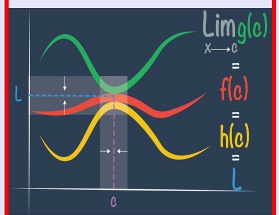
Calculus I

Lecture 41



Feb 19-8:47 AM

Given $f(x) = \frac{x^2}{x^2+3}$

1) Domain $x^2+3 \neq 0$

$(-\infty, \infty)$

2) All asymptotes

NO V.A.

H.A. $y=1$ $\quad \lim_{\substack{x\to\infty \\ x\to-\infty}} \frac{x^2}{x^2+3} = 1$

3) All intercepts

x-Int $(0,0)$ Twice

Y-Int $(0,0)$

4) Even, odd, or neither?

$f(-x) = \frac{(-x)^2}{(-x)^2+3}$

$= \frac{x^2}{x^2+3} = f(x)$

Even Function

Symmetric with respect to Y-axis.

5) Find all Critical pts.

$f'(x)=0$ or undefined

$f'(x) = \frac{2x(x^2+3) - x^2 \cdot 2x}{(x^2+3)^2}$ $\quad f'(x) = \frac{6x}{(x^2+3)^2}$ $\quad (0,0)$

5) Find all possible inflection pts.

$f''(x)=0$ or undefined.

$f'(x) = 6x\,(x^2+3)^{-2}$

Correction

$f''(x) = 6\left[1(x^2+3)^{-2} + x \cdot -2(x^2+3)^{-3} \cdot 2x\right]$

$= 6\left[(x^2+3)^{-2} - 4x^2(x^2+3)^{-3}\right] = 6(x^2+3)^{-3}\left[(x^2+3)^{1} - 4x^2\right]$

$f''(x) = \frac{6(3-3x^2)}{(x^2+3)^3}$ $\quad f''(x)=0$ $\quad 3-3x^2=0$

$x^2=1 \quad x=\pm 1$

P.I.P.

$\underbrace{\left(\pm 1, \frac{1}{4}\right)}_{\text{Make the Correction}}$

$\frac{1}{1+3} = \frac{1}{4}$

Set-up the Sign chart

Discuss increasing/Decreasing

Discuss Concavity

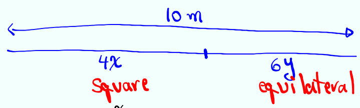
Draw $f(x)$.

Nov 13-7:26 AM

I have a piece of wire 10 m long.

I like to cut it and make a square and equilateral triangle.

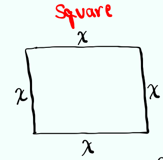
How do I do this to get max. enclosed area or minimum enclosed area.

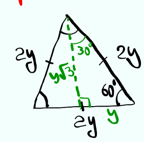


$4x + 6y = 10$

$2x + 3y = 5$

$y = \frac{5-2x}{3}$





$A_{square} = x^2$

$A_{Triangle} = \frac{2y \cdot y\sqrt{3}}{2}$

$= y^2\sqrt{3}$

Total enclosed area

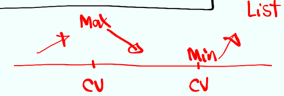
$x^2 + y^2\sqrt{3}$

$$\boxed{f(x) = x^2 + \sqrt{3}\left(\frac{5-2x}{3}\right)^2}$$

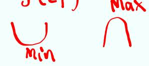
$f'(x)$

$f'(x) = 0$

List of Critical Values



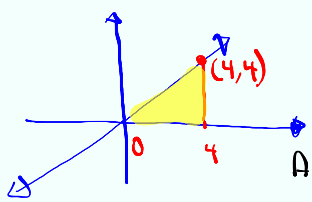
$f''(cp)$



Nov 13-7:46 AM

Find the area below $f(x) = x$, above x-axis from $x=0$ to $x=4$.

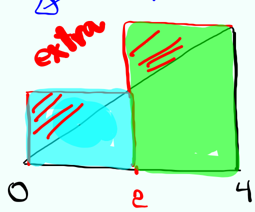
$A = \frac{bh}{2} = \frac{4 \cdot 4}{2} = 8$ No Calculus.

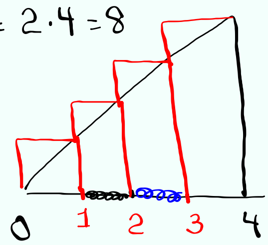


$A_1 = 2 \cdot 2 = 4$

$A_2 = 2 \cdot 4 = 8$

$\to 12$





$A_1 = 1 \cdot 1 = 1$

$A_2 = 1 \cdot 2 = 2$

$A_3 = 1 \cdot 3 = 3$

$A_4 = 1 \cdot 4 = 4$

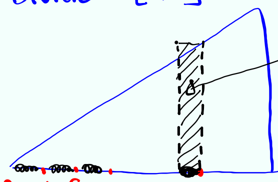
$\to 10$

Nov 13-7:57 AM

Divide $[0,4]$ to n subintervals

$x_i = \frac{4i}{n}$

$f(x) = x$



$A_i = \frac{4}{n} \cdot f(x_i)$

$A_i = \frac{4}{n} \cdot \frac{4i}{n} \qquad A_i = \frac{16i}{n^2}$

$0 \quad \frac{4}{n} \quad \frac{8}{n} \quad \frac{12}{n} \; \ldots \; x_i \; \ldots \; 4$

$0 \quad 4\left(\frac{1}{n}\right) \quad 4\left(\frac{2}{n}\right) \quad 4\left(\frac{3}{n}\right) \quad 4\left(\frac{i}{n}\right) \quad 4\left(\frac{n}{n}\right)$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} A_i = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{16i}{n^2} = \lim_{n\to\infty} \frac{16}{n^2} \sum_{i=1}^{n} i$$

$$\boxed{\underbrace{1+2+3+\cdots+n}_{\sum_{i=1}^{n} i} = \frac{n(n+1)}{2}}$$

$$= \lim_{n\to\infty} \frac{\overset{8}{\cancel{16}}}{n^2} \cdot \frac{n(n+1)}{\cancel{2}}$$

$$= 8 \boxed{\lim_{n\to\infty} \frac{n^2+n}{n^2}} \to 1$$

$$\lim_{n\to\infty} \frac{n^2+n}{n^2} = \frac{\infty}{\infty}$$

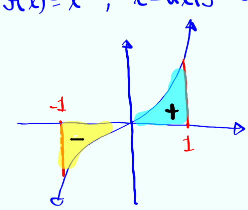
$$\lim_{n\to\infty} \frac{\frac{n^2}{n^2}+\frac{n}{n^2}}{\frac{n^2}{n^2}} = \lim_{n\to\infty} \frac{1+\cancel{\frac{1}{n}}^{\,0}}{1} = \frac{1}{1} = 1$$

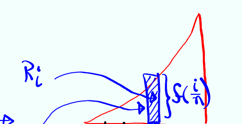
$$= 8 \cdot 1$$

$$= \boxed{8}$$

Nov 13-8:05 AM

Find the area bounded by the graph of $f(x) = x^3$, x-axis from $x=-1$ to $x=1$.





$$A_i = \frac{1}{n} \cdot \left(\frac{i}{n}\right)^3 = \frac{i^3}{n^4}$$

$$\text{Area} = 2 \cdot \lim_{n\to\infty} \sum_{i=1}^{n} A_i$$

$$= 2 \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4} = 2 \lim_{n\to\infty} \frac{1}{n^4} \boxed{\sum_{i=1}^{n} i^3}$$

From PreCalculus:

$1+2+3+\cdots+n = \frac{n(n+1)}{2}$

$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$1^3+2^3+3^3+\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$$= 2 \lim_{n\to\infty} \frac{1}{n^4} \cdot \frac{n^2(n+1)^2}{4}$$

$$= \frac{2}{4} \lim_{n\to\infty} \frac{n^2(n^2+2n+1)}{n^4}$$

$$= \frac{1}{2} \lim_{n\to\infty} \frac{n^4+2n^3+n^2}{n^4}$$

$$= \frac{1}{2} \lim_{n\to\infty} \frac{1 + \cancel{\frac{2}{n}}^{\,0} + \cancel{\frac{1}{n^2}}^{\,0}}{1} = \frac{1}{2} \cdot 1 = \boxed{\frac{1}{2}}$$

Nov 13-8:15 AM

Given $f'(x) = 4 + x^2$ $\quad f(0) = 0$

Find $f(x)$ $\quad f(x) = 4x + \frac{1}{3}x^3 + C$

$f(0) = 4(0) + \frac{1}{3}(0)^3 + C = 0$

$\boxed{C = 0}$

$f(x) = 4x + \frac{1}{3}x^3$

Evaluate $f(2) - f(0)$

$= 4(2) + \frac{1}{3}(2)^3 - 0 = 8 + \frac{8}{3} = \boxed{\frac{32}{3}}$

Nov 13-8:30 AM